**Problem Set 3**

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**Problem 1:**

The solution of EXACT SET COVER must satisfy the following constraint:

(1) For any two sets of a subcollection Y, they cannot have intersection. For instance, W2 and W3 cannot be in Y at the same time, since they both have the same element g:

∀**i, j**  W**i**, W**j** ∈ W, [(∀**x x** ∈ W**i** → x ∉ W**j**)∧ (∀**y y** ∈ W**j** → y ∉ W**i**)]

(2) Each element of U must appear once in subcollection Y. It means that the union of all the sets in Y must be U:

∃ S ⊂ W, ∀W**i** ∈ S, **∪**i=1  |S|  W**i** = U

For instance, Y = {W3, W5, W6}, all of the elements in U appear in Y once.

**Problem 2:**

1. ∀p W(p, u) ⇒ S(p, u)
2. ∀u  [S(B, u) ∧ A(u, H) ⇒ L(B, u)]
3. ∀u,q [F(B, q) ∧ W(q, u) ⇒ S(B, u)]
4. ∀q [F(T, q) ⇒ ∃u [W(q, u) ∧ A(u, H)]
5. ∃q F(T, q) ∧ F(B, q)
6. ∃q ∃u [F(T, q) ∧ W(q, u) ∧ L(B, u)]
7. ∀u W(G, u) ⇒ A(u, H)
8. ∀p F(B, q) ⇒ ∃u W(q, u)
9. ¬ ∃u [W(G, u) ∧ L(B, u)]
10. ¬ F(B, G)

**Problem 3:**

Negation of f: ¬[∃q ∃u [F(T, q) ∧ W(q, u) ⇒ L(B, u)]]

Converted to clausal form:

b. ¬ S(B, u) ∨ ¬ A(u, H) ∨ L(B, u) (1)

c. ¬ F(B, SK**q**) ∨ ¬ W(SK**q**, u) ∨ S(B, u) (2)

d. ¬ F(T, q) ∨ W(q, SK**u**) (3)

¬ F(T, q) ∨ A(SK**u** , H) (4)

e. F(T, SK**q**) (same as (6)) F(B, SK**q**) (5)

¬ f. ¬ [¬ F(T, SK**q**) ∨ ¬ W(SK**q**, SK**u**) ∨ L(B, SK**u**)]

F(T, SK**q**) (6)

W(SK**q**, SK**u**) (7)

¬ L(B, SK**u**) (8)

From (1) and (8), infer ¬ S(B, SK**u**) ∨ ¬ A(SK**u**, H) (9)

Form (2) and (5), infer ¬ W(SK**q** , u) ∨ S(B, u) (10)

From (3) and (6), infer W(SK**q**, SK**u**) (11)

From (7) and (10), infer S(B, SK**u**) (12)

Form (4) and (6), infer A(SK**u**, H) (13)

Form (9) and (13), infer ¬ S(B, SK**u**) (14)

From (10) and (11), infer S(B, SK**u**) (same as (12), can be resolved as same as 12)

From (12) and (14), infer Φ

**Problem 4**

Negation of j : F(B, G) (1)

Converted to clausal form:

b. ¬ S(B, u) ∨ ¬ A(u, H) ∨ L(B, u) (2)

c. ¬ F(B, SK**q**) ∨ ¬ W(SK**q**, u) ∨ S(B, u) (3)

g. ¬ W(G, u) ∨ A(u, H) (4)

h. ¬ F(B, q) ∨ W(q, SK**u**) (5)

i. ¬ W(G, SK**u**) ∨¬ L(B, SK**u**) (6)

From (1) and (5), infer W(G, SK**u**) (7)

From (4) and (7), infer A(SK**u** , H) (8)

From (3) and (5), infer ¬ F(B, SK**q**) ∨ S(B, SK**u**) (9)

From (1) and (9), infer S(B, SK**u**) (10)

From (2) and (10), infer ¬ A(SK**u**, H) ∨ L(B, SK**u**) (11)

From (8) and (11), infer L(B, SK**u**) (12)

From (6) and (7), infer ¬ L(B, SK**u**) (13)

From (12) and (13), infer Φ